Int. J. Heat Mass Transfer. Vol. 15, pp. 367–368. Pergamon Press 1972. Printed in Great Britain

AN IMPROVED RESPONSE EQUATION FOR HOT-WIRE ANEMOMETRY

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(*Received* 28 *April* 1971 *and in revised form* 19 *July* 1971)

1. INTRODUCTION

THE TECHNIQUE of hot-wire anemometry is based upon an empirical law which governs the cooling of an electrically heated wire aligned with its axis normal to the mean flow direction. This law may be written as

$$E^2 = a + b\,U_e^n, \tag{1}$$

where E is the measured voltage drop across the wire when the effective cooling velocity normal to the wire axis is U_e. a, b and n are constants whose values are obtained by representing experimental calibration data by expression (1).

Equation (1) was first derived theoretically by King [1] for the idealised situation of potential flow around an infinitely long cylinder in cross flow, for which the exponent n becomes 0·5, and the equation is known as King's law. If a, b and n are treated as constants, equation (1) can only be used as an accurate representation of calibration data obtained from a hot wire probe in a real flow situation over a very limited velocity range. It can be used to give a very approximate representation over a wide range of velocities (a situation encountered when making measurements in highly turbulent flow fields [2]) by, for example when $n = 0{\cdot}5$, fitting a straight line to the calibration data for E^2 against $U_e^{\frac{1}{2}}$ in the range $0 \leqslant U_e \leqslant (U_e)_{max}$ by either (i) making the line pass through one high velocity point and one low velocity point, or (ii) minimizing the sums of squares of the deviations between the measured and fitted E^2. Neither of these procedures is found to give an adequate representation over the whole velocity range involved when $(U_e)_{max}$ is large. In an attempt to overcome this lack of wide velocity range accuracy, Davies and Bruun [3] suggested using equation (1) with coefficients which are allowed to vary over the velocity range. This procedure, although undoubtedly more accurate, leads to considerable additional labour and complication in its application. The purpose of this note is to describe an alternative method of representing calibration data over a wide velocity range (0–160 ms^{-1}) which is more accurate than any simple linear approximation to E^2 vs. $U_e^{\frac{1}{2}}$ data but which is less complicated to fit and apply than the equations of [3].

2. THE NEW WIRE RESPONSE EQUATION

The wire calibration data shown in Fig. 1 is that given by Davies and Bruun [3] for a 2 mm long tungsten wire, operating at 15 Ω in air at a temperature of 18°C and in the velocity range 0–160 ms^{-1}.

A simple description of this data over the whole flow range (excluding the zero velocity point) is afforded by fitting the second order equation in $U_e^{\frac{1}{2}}$

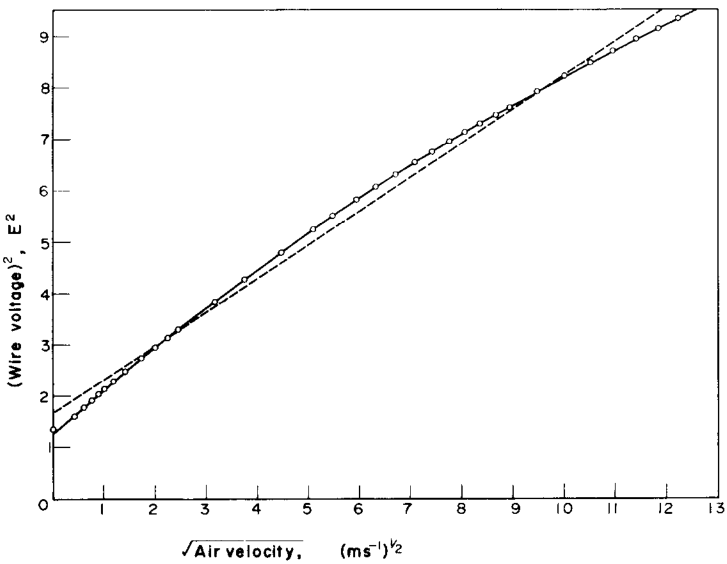
$$E^2 = A + B\,U_e^{\frac{1}{2}} + C\,U_e \tag{2}$$

by minimizing the sums of the squares of the deviations between the measured and the fitted values of E^2, excluding the zero flow point. The full curve in Fig. 1 is the required best-fit to the given calibration data. Also included in the figure for comparison is a dashed line obtained by minimizing the sums of the squares of the deviations between the measured values of E^2 and the values given by equation (1) with $n = 0{\cdot}5$. The constants characterizing the curves are given in the figure for information. Inspection of the data presented in the figure indicates that equation (2) produces a significantly better representation of the calibration data than equation (1). The standard error of estimate (Spiegel [4]) of E^2 on $U^{\frac{1}{2}}$ for equation (2) is 0·027 whilst for equation (1) it is 0·311.

The addition of an extra velocity term to King's law therefore produces a significant improvement in the accuracy attainable by normal anemometry over a wide velocity range.

3. SIGNAL INTERPRETATION WITH THE NEW EQUATION

The conventional method of interpreting anemometer responses, based on equation (1) with $n = 0{\cdot}5$, leads to an



Values of constants

King's law		Quadratic form		
a	*b*	A	B	C
1·644	0·656	1·273	0·860	—0·017

FIG. 1. Alternative representations of the hot-wire calibration data of Davies and Bruun [3].

expression for the mean flow velocity normal to the wire axis ($\bar{U}$) of the form

$$\bar{U} = \left\{\frac{(\bar{E})^2 - a}{b}\right\}^2, \tag{3}$$

and to an expression for the intensity of turbulence in the same direction of the form

$$\frac{[(\overline{U'^2})]^{\frac{1}{2}}}{\bar{U}} = \frac{4\bar{E}[(\overline{E'^2})]^{\frac{1}{2}}}{(\bar{E})^2 - a}. \tag{4}$$

Repeating the analysis leading to these results using equation (2) produces the formulae

$$\bar{U} = \left\{\frac{D^{\frac{1}{2}} - B}{2C}\right\}^2 \tag{5}$$

and

$$\frac{[(\overline{U'^2})]^{\frac{1}{2}}}{\bar{U}} = \frac{4\bar{E}[(\overline{E'^2})]^{\frac{1}{2}}}{\left\{\frac{D - BD^{\frac{1}{2}}}{2C}\right\}} \tag{6}$$

where $D = B^2 + 4C[(\bar{E})^2 - A]$.

Note that equations (5) and (6) reduce to (3) and (4) as $C \to 0$, with A and B replacing a and b. The use of equations (5) and (6) instead of (3) and (4) represents very little additional computational effort.

4. CONCLUSIONS

The description of the behaviour of a normal hot-wire anemometer over a wide velocity range has been shown to be significantly improved by the addition of an extra term to the conventional King's law. This addition does not lead to any substantial increase in the difficulty of interpretation of anemometer signals but it does produce improved accuracy in the derived values of $\bar{U}$ and $(\overline{U'^2})^{\frac{1}{2}}/\bar{U}$. The new method has already been successfully applied to measurements made in highly turbulent flow fields [2].

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